# AP Calculus AB Syllabus

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| **CR1** | The students and teacher have access to a college-level calculus textbook, in print or electronic format. | See page: 2 |
| **CR2** | The course is structured to incorporate the big ideas and required content outlined in each of the units described in the AP Course and Exam Description. | See page: 2, 3 |
| **CR3** | The course provides opportunities for students to develop the skills related to Mathematical Practice 1: Implementing Mathematical Processes. | See page: 6, 7, 8, 9, 10 |
| **CR4** | The course provides opportunities for students to develop the skills related to Mathematical Practice 2: Connecting Representations. | See page: 5, 6, 7, 8, 10, 11 |
| **CR5** | The course provides opportunities for students to develop the skills related to Mathematical Practice 3: Justification. | See page: 5, 8, 11 |
| **CR6** | The course provides opportunities for students to develop the skills related to Mathematical Practice 4: Communication and Notation. | See page: 8, 10 |
| **CR7** | Students have access to graphing calculators and opportunities to use them to solve problems and to explore and interpret calculus concepts. | See page: 2, 3, 4, 5, 7, 10, 11 |
| **CR8** | The course provides opportunities for students to use calculus to solve real-world problems. | See page: 3, 8, 11 |

## Course Description

AP® Calculus AB is a yearlong course that is comparable to a semester long college-level course designed to prepare students for the Advanced Placement (AP) Calculus AB exam. Major topics of study in this full-year course include a review of pre-calculus functions; the use of limits, derivatives, definite integrals, mathematical modeling of differential equations; and the applications of these concepts. Emphasis is placed on notational fluency between the big ideas, the different representations, and the strategic use of technology to solve problems and draw conclusions. The course uses a multi-representative approach to calculus, with concepts and problems expressed numerically, graphically, verbally, and analytically. This course is aligned to the new College Board AP Calculus AB course description that was introduced in 2020.

## Classroom Requirements

This AP course requires that a highly qualified math teacher be responsible for administering the course. This teacher is expected to interact with students regularly (either face to face or via chat and e-mail) and grade open-ended student responses. The teacher also should set up online discussions in the Collaboration Corner. In addition, he or she should provide instruction and coaching in calculator use as needed.

# Course Materials

## Textbook

Franklin D. Demana, Bert K. Waits, Daniel Kennedy, David M. Bressoud, and Michael Boardman. *Calculus: Graphical, Numerical, Algebraic*, 6th ed. Boston: Pearson, 2020. **[CR1]**

## Graphing Calculator

On-screen teachers will use an online graphing calculator tool for demonstration purposes. **Students must have access to an AP-approved graphing calculator both in the classroom and at home by purchasing one of their own or by borrowing one from the school.** Many assignments, projects, and practice exam questions will require the use of an AP-approved calculator. Supplemental information regarding use of graphing calculator software or a graphing calculator will also be provided in each unit.

Classroom teachers are expected to demonstrate the use of graphing calculators to their students as needed. **[CR7]**

**Examples of Graphing Calculator Activities**

### Graph functions

Technology Corner: Every technology corner in each unit has students using their graphing calculators to solve problems and explore the big ideas of that unit. **[CR7]**
**Sample of evidence:** Students are given a worksheet in the Unit 8 Technology Corner with two functions defined analytically. They are asked to use their graphing calculator to graph the functions and determine the points of intersection. They then must use integration capabilities of the graphing calculator to determine the sum of the areas created by the intersecting graphs.

### Solve equations

Reading Lessons 1.1 and 8.2 (Areas in the Plane): Students graph linear functions using their calculators, and utilize the calculator's equation solver capabilities to determine the point of intersection in Reading Lesson 1.1.1 Students revisit their system of equations calculator skill set to determine points of intersection between two or more equations in order to graph the area enclosed by intersecting curves and to determine the endpoints needed for integration and then use the calculator's numerical integration function to find the area. 2 **[CR7]**

### Perform numerical differentiation

Differentiation Rules: In this lesson, students use the power rule to find derivatives, the product rule to find derivatives, and the quotient rule to find derivatives. Students calculate second derivatives and higher-order derivatives using rules of differentiation. Students also calculate instantaneous rates of change at particular $x$-values using their graphing calculator's numerical differentiation. **[CR7]**

### Perform numerical integration

Trapezoidal Rule: In this lesson, students are to approximate the area under a curve using the trapezoidal rule using their graphing calculators. **[CR7]**

**Unit 6 Free Response Questions** Students are given various functions that they cannot integrate analytically using the techniques they have learned. They must instead use the graphing calculator's numerical integration feature to compute definite integrals of these functions on stated intervals. **[CR7]**

### Explore or interpret calculus concepts

Across all lessons in the course, students are required to use their graphing calculator to solve problems, especially in the Technology Corner projects. Students will use their graphing calculators experimentally to determine possible solutions to problems. They will also learn to use their calculators to justify their conclusions and to support the solutions that they have developed using an analytical approach.
Newton's Method, Linearization, and Differentials: In this lesson, students are asked to apply Newton's method to approximate zeros of a function and to use linearization to approximate tangent lines. **[CR7]**

## Approach

Students are expected to utilize multiple approaches for understanding the course content. Each unit of the course contains a concise overview of the key definitions, theorems, and concepts. Lessons throughout the units have instruction followed by a task to assess students' understanding and indicate misconceptions and example problems aligned to specific AP reasoning. These assessments are done both with and without graphing calculator use. **[CR7]** Units also contain 1-2 conceptual projects to assess students' ability to make connections across definitions and theorems and allow them to demonstrate notational fluency as well as justifications for their work. **[CR2]** Students will complete unit tests that mimic the multiple-choice AP exam format as well as free response questions at the end of each unit. **[CR8]** The "Technology Corner" in each unit demonstrates the key concepts of each unit using graphing technology and allows students to practice using technology to justify their reasoning and understanding. Students will have to use their graphing calculator or Desmos to solve problems. **[CR7]** Each unit will also contain specific directions for using the TI Nspire CAS graphing calculator. Students will also use the Collaboration Corner feature in the course to facilitate discussions with their teacher and/or with peers. The course concludes with two full-length practice exams and extensive review of the big ideas and mathematical practices.

## Unit 1: Pre-Calculus Review

### Big Ideas and CED Alignment:

Pre-Calculus Review covers key concepts from Pre-Calculus which are necessary to be successful in AP Calculus AB. This unit is not aligned to the CED units or Big Ideas.

## Unit 2: Limits and Continuity

### Big Ideas and CED Alignment:

Limits and Continuity covers required course content from Unit 1 in the CED, including the Big Ideas of Limits, Change, and Analysis of Functions.

## Unit 3: Derivatives

### Big Ideas and CED Alignment:

Derivatives covers required course content from Unit 2 in the CED, including the Big Idea of Limits, Change, and Analysis of Functions.

## Unit 4: More Derivatives

### Big Ideas and CED Alignment:

More Derivatives covers required course content from Unit 3 in the CED, including the Big Ideas of Analysis of Functions.

## Unit 5: Applications of Derivatives

### Big Ideas and CED Alignment:

Applications of Derivatives covers required course content from Units 4* and 5 in the CED, including the Big Ideas of Limits, Change, and Analysis of Functions.

*Straight-line motion is covered in Unit 3, and L'Hopital is covered in Unit 8.

## Unit 6: Definite Integrals

### Big Ideas and CED Alignment:

Definite Integrals covers required course content from Unit 6* in the CED, including the Big Ideas of Limits, Change, and Analysis of Functions.
*Concepts of integration using substitution are found in Unit 7.

## Unit 7: Mathematical Modeling Using Differential Equations

### Big Ideas and CED Alignment:

Mathematical Modeling Using Differential Equations covers required course content from Unit 7 in the CED, including the Big Idea of Analysis of Functions.

## Unit 8: Applications of Definite Integrals

### Big Ideas and CED Alignment:

Applications of Definite Integrals covers required course content from Unit 8 in the CED, including the Big Idea of Change.

# Course Outline

### Unit 1 – Pre-Calculus Review

Big Ideas: Analysis of Functions

#### *Topics for Overview*

- Intro to AP Calculus
- Linear Functions
- Functions and Graphs
- Exponential Functions
- Parametric Equations
- Inverse Functions and Logarithms
- Trigonometric Functions

#### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 0) **[CR1]**

#### *Examples of Key Activities*

Students will review key functions and understandings that are prerequisite for the course. In the Technology Corner students will explore how to adjust function tables and graphs in their calculators and with graphing technology. Students will also review functions with domain restrictions, vertical asymptotes, or horizontal asymptotes using graphing technology or their calculators. **[CR7]**

### Unit 2 – Limits and Continuity

Big Ideas: Change, Limits, Analysis of Functions

### *Topics for Overview*

- Introduction to Limits and Continuity
- Rates of Change, Limits, and the Squeeze Theorem
- Limits Involving Infinity and Vertical and Horizontal Asymptotes
- Continuous Functions and the Intermediate Value Theorem
- Slope, Tangent Line, and Normal Line

### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 1) **[CR1]**

### *Examples of Key Activities*

Students will begin with a comparison between average rates of change and instantaneous rates of change with the context of distance/rate/time scenarios. Students will learn the definition of limit and the situations where discontinuities exist, focusing on both the symbolic and written form. Students will find limits across multiple representations, including the concept of when the limit exists but the function value does not, and vice versa. Students will use technology to explore the essential conditions of and the applications of the Squeeze Theorem and The Intermediate Value Theorem, identifying mathematical information and the appropriate mathematical procedure given graphically, symbolically, numerically, and verbally. **(Skill 3.C, 3.F) [CR4]**

Students will use technology and analytical methods to find limits at infinity and horizontal asymptotes. **[CR7]** Students will begin exploring the equations of secant, tangent, and normal lines. In the Technology Corner students will determine the appropriate method for determining the limit of a given function, and practice computing limits with algebraic rules, graphically, and with a table of values to analyze the behavior of f for values of x using technology. Students will explore the limit of composite functions, using the graph to justify the value of the limit from one direction of the inside function determining the limit of the outside function. Students will have to use multiple representations to find a limit, including a graph, two functions, and a table of values. For example: given a graph, students will write the analytical form of the limit using correct notation; given the analytical form of a limit of a function, students will use a table of values to determine the limit. **(Skill 2.C) [CR4]; [CR5]**

In the project for this unit students will confirm that initial conditions are met to utilize the Squeeze Theorem and then use technology to explain their solutions, specifically utilizing a table of values and graphs to justify their conclusions. Students will be given the function $f(x) = x^2 e^{\cos\frac{1}{x}}$ analytically and be asked to find the limit by determining functions that will bound $f(x)$ and providing justifications for their choices. **[CR5]** They will also continue this justification, exploration, and solution for the Intermediate Value Theorem using a piecewise function. **(Skill 4.A)** Students will create examples and counterexamples to clarify understanding and investigate whether converses of theorems are true or false, or to test conjectures using technology. **[CR5]; [CR7]**

## Unit 3 – Derivatives

Big Ideas: Change, Limits, Analysis of Functions

### *Topics for Overview*

- Introduction to Derivatives
- Derivatives of Functions
- Derivatives and Continuity
- Differentiation Rules
- Applications of Derivatives
- Differentiating Trigonometric Functions

### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 2) **[CR1]**

### *Examples of Key Activities*

Students will explore the transformation of the secant line to the tangent line by reducing the value of h (the distance between x values) while simultaneously paying attention to the rates of change of both lines as the transformation occurs using technology. Students will be connecting the ideas of continuity and differentiability by investigating functions where one exists and the other does not, specifically learning that continuity is a requirement for differentiability but not a guarantee that the derivative will exist. Students will justify their reasoning using graphs, symbolic notation, and/or verbal representations with and without technology. The instruction and assessment items will utilize different notation structures to give students the opportunity to develop notational fluency, especially related to higher order derivatives. Students will determine the appropriate rule for determining a derivative and provide rationale for their conclusions. **(Skills 1.D and 1.E)**

Students will connect derivatives to the real-world application of position and velocity or velocity and acceleration. Students will investigate the graphical relationships between given functions and their derivatives both by hand and using technology. **(Skill 2.B, 2.D, 2.E)**

In the Technology Corner students will explore an interactive that transforms the secant line into a tangent line while displaying the rates of change simultaneously. Students will then answer questions regarding how the limit as h approaches zero results in the two rates of change becoming closer to equality and give rationale for any possible errors. Students will document their exploration with appropriate notation and terminology.

In the unit project students will explore graphs of discontinuous functions and continuous functions with regards to when differentiability exists and does not exist; specifically functions where two-sided limits exist but the function value does not, functions where the value at a point exists but the limit does not, and functions that are continuous but have points where the derivative does not exist. They will be required to articulate their justifications for problem solutions with correct terminology and notation. **[CR3; CR4]** For example, students will be given a piecewise function and be asked to justify if the limit and derivative exist for a particular value. Additionally, students will explore a function rule defined by operations of another function with values given in a table of values only. They will use specific derivative notation and the Intermediate Value Theorem to justify the existence of a zero over a given interval. **(Skill 2.D)** Students will also use a table of values for a function and its derivative, as well as a linear piecewise function's graph to determine the derivative of a product of those two functions, using the correct notation for the product rule. **[CR 8]**

## Unit 4 – More Derivatives

Big Ideas: Change, Analysis of Functions

### *Topics for Overview*

- Introduction to More Derivatives
- Differentiating Functions Using the Chain Rule
- Differentiating Functions Using Implicit Differentiation
- Differentiating Functions Containing Inverse Functions and Inverse Trigonometric Functions
- Differentiating Exponential and Logarithmic Function

### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 3) **[CR1]**

### *Examples of Key Activities*

Students will be given a set of functions defined analytically and they will be asked to first determine which are composite functions and secondly, use the chain rule to determine the derivative. **[CR3]** Students will investigate nonfunctional relationships and choose the appropriate procedure for finding the derivative implicitly. Students will use technology to explore the derivatives of the inverse of functions given symbolically or as a table of values and explain their reasoning and justify their solution path. Students will analyze exponential and logarithmic functions with and without technology to choose the appropriate rule for calculating derivatives.

In the Technology corner students will explore inverse functions with graphing technology to see the relationship between the domain and range. Students will have to use their graphing calculators or Desmos to graph a function and its inverse in the same window, and they will have to use technology to determine the slope of the tangent line for a function they cannot take the derivative of by hand, $f(x) = \arcsin\frac{x}{2}$. Students will be asked to graph a derivative given a graph of an initial function, indicating all the key characteristics. **[CR7]**

In the project for this unit, students will explore the chain rule and composition of functions with multiple models, including decomposing functions. For example, students will be given a function rule and a piecewise graph. They will then be asked to determine the equation of the tangent line at a point of a function that is the composition of the two given functions. Their solution will have to use the correct derivative rule and notation and provide the correct algebraic steps. Students will outline the appropriate sequence of algebraic steps for explicit and implicit derivatives, and then complete those steps. **[CR3; CR4]**

## Unit 5 – Applications of Derivatives

Big Ideas: Change, Analysis of Functions

### *Topics for Overview*

- Introduction to Applications of Derivatives
- Relative and Absolute Extrema
- The Mean Value Theorem

- The First and Second Derivative Tests
- Application Problem-Solving
- Newton's Method, Linearization, and Differentials
- Applications of Implicit Differentiation

### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 4) **[CR1]**

### *Examples of Key Activities*

Students will solve application problems by determining and applying the appropriate derivative rule based on the contextual relationship and explain their solutions in context. Students will explore the Extreme Value Theorem with and without technology, for both continuous and discontinuous functions, and provide rationale for the conditions of the theorem and when functions will still have extreme values even if it is not continuous over a closed interval. **(Skill 3.C, 3.E)**

Students will analyze the graph of a derivative to determine key features of the original function such as intervals of increasing or decreasing, local extrema, points of inflection and intervals of upward or downward concavity. Students will focus on correct notion when performing the First and Second Derivative Tests. **(Skill 2.E)**

Students will calculate approximations when applying Newton's Method for linearization and relate their solutions to the actual value with and without technology. Students will choose the appropriate implicit derivative methods for solving related rate problems and then justify their answers in context. **(Skills 1.D, 1.E)**. Students will provide step-by-step written solutions, graphs sketched by hand, and justifications for their solution to achieve the can's optimization. **(Skill 2.D)**

Students will connect the big ideas of position, velocity and acceleration with first and second derivatives. For example, students will have to use their understanding of rates of change and derivatives to solve a problem regarding water treatment input and output rates, estimating the critical values and using those estimates to determine maximum and minimum water storage changes in the dynamic system.

In the Technology Corner students will explore linearization with graphing technology and apply Newton's Method to more complex functions. Specifically, students will use their graphing calculators or Desmos to perform three iterations of Newton's Method to estimate a zero of a given function. **(Skill 2.D)** Students will also have to use the Mean Value Theorem to justify the existence of a specific function value over an interval, using correct notation and appropriate terminology in their answers. **(Skill 4.A)** Students will focus on the correct function notation and justifications for when the approximations are valid and when they are misleading. **[CR8]**

In one of the projects for this unit students will solve an optimization problem to find the most economical dimensions for a soda can, given a specific volume that the can must hold. Students will provide step-by-step written solutions, diagrams, and justifications to all the problems to be solved to achieve the can's optimization. **[CR5]** In addition, students will conduct research to identify real-world examples and non-examples of their optimized can design and write a report about their findings. Students will present their findings to the class and the teacher will assess their use of appropriate mathematical language. **[CR3]; [CR6]; [CR8]**

## Unit 6 – Definite Integrals

Big Ideas: Change, Limits, Analysis of Functions

### *Topics for Overview*

- Introduction to Definite Integrals
- Estimating with Finite Sums
- Definite Integrals
- Definite Integrals and Antiderivatives
- The Fundamental Theorem of Calculus (Parts I and II)
- The Trapezoidal Rule

### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 5) **[CR1]**

### *Examples of Key Activities*

Students will practice estimating with finite sums and finding definite integrals and antiderivatives by connecting the ideas of velocity and position. They will also calculate the areas under curves using various techniques, including rectangular approximation methods (Left, Right, Midpoint) and the trapezoidal rule. Students will be asked to identify which summation technique will result in underestimations or overestimations and present their results to their teacher and/or their classmates. **[CR3]** In addition, students will calculate the average value of a function over a closed interval and use this concept to understand the Mean Value Theorem. Students will interpret the area under a graph as a net accumulation of a rate of change. Students will utilize Riemann sum notation in terms of taking limits of sum with the number of subdivisions tending to infinity and reinterpret those sums using integral notation. **(Skill 4.C)** Students will explore the connections between the limit of the sum of infinite rectangles to the area under the curve given by integrals using technology and write their solutions using appropriate mathematical symbols. **(Skills 2.B, 2.D)** Students will apply the first part of the Fundamental Theorem of Calculus to solve problems and the second part to calculate integrals analytically. **(Skill 3.E)** Students will select the appropriate rule or procedure for finding definite and indefinite integrals of given functions.

In the Technology Corner students will use graphing technology to explore the transition from an infinite sum of rectangles to the exact area under the curve. They will start with a set number of rectangles, observe overestimates or underestimates, increase the number of rectangles, make more observations, and then predict the final area. Students will be given a function analytically and then must rewrite an infinite sum into definite integral notation and to rewrite a definite integral as a sum of infinite rectangles with a focus on the correct notation in both problems. **(Skill 2.C) [CR 4]**

In the project students will analyze a table of velocity data to determine distance traveled. This will involve students correctly representing the distance using both limit notation and integral notation. Students will also use different Riemann Sums to calculate the distance and compare and contrast the best method based on the nature of the data. Students will also use the Fundamental Theorem of Calculus to analyze an integrally defined function given a graph of the integrand. **[CR 8]** Students will be expected to determine and justify behaviors of the function using the graph, such as increasing/decreasing, concavity, local extrema, points of inflection by making the connection between the integrally defined function and the graph. That is, students will use the connections between $g' = f$ and $g'' = f'$ given that $g(x) = \int_a^x f(t)dt$ to solve problems involving $g(x)$ and the intervals of increase or decrease and intervals of concavity changes and points of maximum and minimum values. **(Skill 2.C)** Students will also be expected to solve problems using the graph such as finding the equation of the tangent line for the function and using L'Hospital's Rule to evaluate a limit involving the integrally defined function. **[CR3]; [CR4]; [CR6]; [CR7];**

## Unit 7 – Mathematical Modeling Using Differential Equations

Big Ideas: Change, Analysis of Functions

### *Topics for Overview*

- Introduction to Mathematical Modeling Using Differential Equations
- Slope Fields
- Antidifferentiation by Substitution
- Exponential Growth and Decay

### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 6) **[CR1]**

### *Examples of Key Activities*

Students will model real-world rates of change using differential equations using appropriate notation and solving with the necessary procedures. Students will extend their understanding of rates of change by identifying slope fields to represent solutions to differential equations and sketch slope fields using technology and by hand. **(Skill 2.D)** Students will explain their graphs using the correct notation so that all possible solutions are represented when an initial condition is not given, and how their answers would be adjusted given an initial condition. **(Skill 4.A)** Students will confirm that solutions to differential equations are accurate and appropriate. Students will utilize the chain rule for the integration of composite functions using substitution. Students will select the appropriate antidifferentiation technique to solve differential equations involving exponential growth and exponential decay.

In the Technology Corner students will graph slope fields with their graphing calculators or Desmos. They will critically interpret the meaning of the graphs to both general and specific solutions to differential equations using rate of change language and accurate notation. They will need to graph a solution given a slope field. Students will make connections from the differential equation to its corresponding slope field and then to its general solution and the particular solution, including matching slope fields to differential equations. **(Skill 2.C)**

In the unit's project students will develop conjectures on antidifferentiation using substitution and the relationship to the chain rule by identifying the inner function and its derivative, then correlating this with the integration using substitution. Students will be asked to identify multiple functions that are solutions to a given differential equation and they will be asked to determine a parameter of a function so that it will be a solution to a given differential equation. **(Skill 2.C) [CR3]; [CR6]**

## Unit 8 – Applications of Definite Integrals

Big Ideas: Change, Limits, Analysis of Functions

### *Topics for Overview*

- Introduction to Applications of Definite Integrals
- Integral as Net Change
- Areas in the Plane
- Volumes
- Applications from Science and Statistics
- L'Hospital's Rule and Other Applications

### *Textbook Reading*

- Demana et al, 2020 (selections from chapter 7) **[CR1]**

### *Examples of Key Activities*

Students will practice applying the definite integral to solve problems involving accumulation, displacement, total distance, net change, areas, work, fluid pressure, and probability distribution, and justify their findings in writing using appropriate notation. Students will calculate the volume of a solid generated by revolving a line or curve around a given line and by revolving a region bounded by two or more lines or curves. They will also calculate the volume of a solid using cross-sections.

Students will practice using L'Hospital's Rule to evaluate limits of indeterminate forms and compare the growth rates of functions. Students will connect the different representations using correct notation to understand how L'Hospital's provides the same result as other limit procedures. **(Skill 3.C, 3.E) [CR4]**

In the Technology Corner students will using graphing technology to explore the changes in the area function to be integrated as the rotation moves off the axes (both horizontal and vertical). Students will conjecture a possible generalization in notation for the inner and/or outer radius for calculating volume. Students will be asked to find the sum of areas created by two graphs intersecting by using technology to graph the functions and estimate the intersection values using graphing calculators or Desmos. Students will then consider the solid created by cross sections for an area but will need to use their calculators to evaluate the integral created. Students will also calculate the volume of a solid created by rotating two functions about the vertical line $x = 1$.

In the unit's project students will explore the concept of volume with uniquely shaped vases generating a table of values comparing the height and the radius at different intervals and then creating a piece-wise function by hand to determine the equation of the function that would be used to create the vase using rotation about a line. Students will then measure the volume of the vase and compare it to the volume created by their equation and explain any differences using appropriate notation and terminology. Students will also need to determine a parameter value that will create the maximum cross-sectional area with a given radius. **[CR5]; [CR7]; [CR8]**

## Unit 9 – Course Review and AP Exam Preparation

Big Ideas: Change, Limits, Analysis of Functions

## *Topics for Overview*

The course concludes with a unit focused on preparing students for the AP exam with a review of the big ideas covered in each unit and two practice exams. **[CR 8]**